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An Analytical Approach for the Optimal Operation of Simultaneous AC-DC Power Transmission System

Mohammad Tawhidul Alam[1]and Q. Ahsan[2]

[1]Department of Electrical and Electronic Engineering, Daffodil International Univeristy, Bangladesh
[2]Department of Electrical and Electronic Engineering, Green University of Bangladesh, Bangladesh

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ABSTRACT

Simultaneous AC-DC power transmission technique can improve both loadability and stability of a power system with long transmission line. But, there is a tradeoff between loadability and stability, i.e. increase in the improvement of loadability causes the decrease in the improvement of stability and vice versa. Actually, it is a multi-objective optimization problem where the objective function depends on two decision variables with opposite in nature. Firstly, this paper presents an analytical expression for the objective function which is the function of two decision variables; power flowing capacity and critical clearing time. Secondly, a mathematical model is developed for the optimal point of the objective function. Considering a typical system a numerical analysis is performed using the proposed expressions. Again, the impacts of the line length and the voltage level of a transmission line on the objective function are also investigated. Finally, the developed model of the optimal point is validated to judge its accuracy and applied to a real system to justify its ability to evaluate the combined benefit of loadability and stability of simultaneous AC-DC system.

Corresponding Author:

Mohammad TawhidulAlam,
Department of Electrical and Electronic Engineering,
Daffodil International Univeristy,
Dhaka, Bangladesh.
Email: drtawhidul.eee@diu.edu.bd

1. INTRODUCTION

Load carrying capability of a long EHV transmission line is mainly limited by the steady state stability [1-2]. Moreover, it is required to keep sufficient margin against the transient stability. To improve the loadability and stability of an existing AC line present practices are the use of series capacitive compensation [3-5] and FACTS devices [6-8]. A new approach of power transmission, where DC is allowed to flow through the AC line simultaneously, can be a solution to improve the performance of existing transmission system. The main benefit of simultaneous AC-DC system is that it can improve both loadability [9-11] and stability [12-14].

An appropriate mathematical model is developed in [15] for the loadability analysis of simultaneous AC-DC system. In this model, AC and DC power of AC-DC system are expressed in terms of AC power flow of original AC system. The work of the paper clearly shows that the higher amount of power flow cannot be achieved in all operating conditions. Using the mathematical model, the limiting values of the variables of any system can be determined for which the improvement in loadability is just marginally possible. From the numerical analysis of [15] it is seen that the power flow increases with the increase of voltage mix and transmission angle.

An elaborate numerical analysis of power flow through simultaneous AC-DC system is presented in [9]. This analysis shows the power flow variation with the change of transmission angle for a fixed DC voltage mix. It is observed that the power flow increases with the increase of transmission angle at a certain level and beyond which power flow decreases with the increase of transmission angle.

A mathematical model for the stability analysis of simultaneous AC-DC system is shown in [16]. The model is based on the equal area criterion of stability analysis and it can handle only the severe most fault of transmission line. Stability analysis of simultaneous AC-DC system is a bit complex task. To reduce this complexity, K.P Basu [17] proposed a new approach where the AC-DC composite system is treated as a pure DC system by turning off the circuit breakers of AC power flow of this system as soon as the fault is cleared.

H. Rahman and B.H. Khan [12] presented a clear-cut comparison of stability between the simultaneous AC-DC transmission line and series compensated original AC transmission line. They have shown that the simultaneous AC-DC system is better than series compensated AC system in stability point of view.

Although simultaneous AC-DC system can improve loadability and stability it is also found that all the works in the literature either on loadability or on stability improvement. But, no work has yet been found on the combined improvement. Therefore, it is necessary to find out an optimal operating point of a system where the loadability and stability both can be improved at a reasonable level.

The problem where the ultimate goal is a function of more than one variable of conflicting in nature is called multi-objective problem. A solution of radio frequency (RF) circuit sizing optimization has been presented in [18] where the problem is subdivided into two parts. One is the normalization of the objective function and other one is the assignment of weights to the objectives. Multi-objective optimization problems have many Pareto solutions and decision is made among them cosidering the total balance over all the objectives taking into account. The totally balancing over criteria is usually called trade-off. H. Nakayama et al. discussed the different approaches of trade-off in [19]. It also mentioned the difficulty in weighting method and provides a way to overcome this difficulty.

In solving multiobjective problems, decision maker may be interested in a set of Pareto optimal points instead of single point. Genetic algorithm (GA) can be used to solve multi-objective problems as it works with a population of points. N. Srinivas and K. Deb [20] investigated Goldbergs's notation of non dominated sorting in GAs along with niche and specification method to find multiple pareto-optimal points simultaneously. The performance of evolutionary algorithm (EA) and conventional gradient based method are demonstrated in [21] for finding Pareto fronts. It also shows the application of multi-objective algorithm in an analytical test problem as well as real-world problems.

It is evident that the higher the improvement in loadability the lower the improvement in stability and vice versa. Therefore, the loadability and stability improvements at a time is a multi-objective problem. This paper presents an analytical model for the combined improvement of loadability and stability of simultaneous AC-DC system through weighted sum multiobjective optimization approach.

2. PROPOSED MODEL

The main purpose of simultaneous AC-DC system is to improve loadabiliy and stability both. The equations (1) and (3) present the expressions of loadability [15] and stability [16], respectively.

$$P_{comb} = \left[(1-k)\beta + \sqrt{2}\,k\gamma\right]P_l \tag{1}$$

$$or \quad f_1 = \left[(1-k)\beta + \sqrt{2}\,k\gamma\right]P_l \tag{2}$$

$$T_{CR} = T_{cr}\sqrt{\left(1 - \frac{P_{comb}}{\left(P_{DCpf} + \bar{P}_{acm}\right)}\right)\left(\frac{\delta_m - \delta_{ac}}{\delta_{cr} - \delta_0}\right)\frac{P_l}{P_{comb}}} \tag{3}$$

$$or \quad f_2 = T_{cr}\sqrt{\left(1 - \frac{P_{comb}}{\left(P_{DCpf} + \bar{P}_{acm}\right)}\right)\left(\frac{\delta_m - \delta_{ac}}{\delta_{cr} - \delta_0}\right)\frac{P_l}{P_{comb}}} \tag{4}$$

Where, P_{comb} and T_{CR} are represented by f_1 and f_2 in equations (2) and (4) respectively.

Generally loadability, f_1, and stability, f_2, are reciprocal in nature. That is, if a system is operated at higher loadability the stability margin will be lower. It is desirable to maximize both the objectives but due to trade off it is not possible to get both of them at their highest levels. There are several methods available for solving this kind of multi-objective optimization problem and among them weighted sum approach is used in this work [18-19]. As the stability and loadabiliy both are to be increased the objective function Z may be expressed as

$$Z = \sum_{i=1}^{N} W_i \bar{f}_i \tag{5}$$

$$Z^* = Max \sum_{i=1}^{N} W_i \bar{f}_i \tag{6}$$

Subject to,

$$\sum_{i=1}^{N} W_i = 1$$

$$W_i > 0$$

$$\bar{f}_i^{(L)} \leq \bar{f}_i \leq \bar{f}_i^{(U)}$$

Where,

N = Number of decision variables, $i = 1, 2, ...N$
W_i = Relative weight of i_{th} decision variable
$\bar{f}_i$ = Normalized value of i_{th} decision variable f_i
Z^*= Maximum value of the objective function
$f_i^{(L)}$ and $f_i^{(U)}$ indicates the upper and lower bound of the i_{th} decision variable.

As there are only two objectives in this work i.e. loadability and stability the value of N is 2 and the objective function Z becomes

$$Z = W_1\bar{f_1} + W_2\bar{f_2} \tag{7}$$

Where,

W_1 = Relative weight for loadability
W_2 = Relative weight for critical clearing time (stability)

The normalization of the decision variables can found as

$$\begin{aligned}\bar{f_1}(k,\beta,\gamma) &= \frac{f_1 - f_1^{min}}{f_1^{max} - f_1^{min}} \\ &= \bar{P} \\ &= \frac{P_{comb} - P_{min}}{P_{max} - P_{min}}\end{aligned} \tag{8}$$

$$\begin{aligned}\bar{f_2}(k,\beta,\gamma) &= \frac{f_2 - f_2^{min}}{f_2^{max} - f_2^{min}} \\ &= \bar{T} \\ &= \frac{T_{CR} - T_{min}}{T_{max} - T_{min}}\end{aligned} \tag{9}$$

Where,

$\bar{P}$ = Normalized value of Loadability
$\bar{T}$ = Normalized vale of critical clearing time

The equation (3) for T_{CR} can be written in the following simplified form

$$T_{CR} = M\sqrt{\left(\frac{1}{P_{comb}} - \zeta\right)} \tag{10}$$

Where,

$$M = T_{cr}\sqrt{\left(\frac{\delta_m - \delta_{ac}}{\delta_{cr} - \delta_0}\right) P_l}$$

$$\zeta = \frac{1}{\left(P_{DCpf} + \bar{P}_{acm}\right)}$$

Now the equation (9) becomes

$$\bar{f_2} = \frac{M\sqrt{\left(\frac{1}{P_{comb}} - \zeta\right)} - T_{min}}{T_{max} - T_{min}} \tag{11}$$

To evaluate the weighted sum of this optimization system the relative weight approach is utilized. Relative weights reflect the relative importance of the objectives. For example, if 1 unit gain in objective-1 can be compensated by two unit lose in objective-2, then the relative weight of objective-1 will be 2 while the relative weight of objective-2 will be 1. If the objectives are normalized and summation of all the weights is 1(one) then the relative weights must be out of 1(one). The calculation of relative weights for stability and loadability are elaborately shown below.

$$W_2 = \frac{1}{1 - \frac{d\bar{T}}{d\bar{P}}} \tag{12}$$

Where, $\frac{d\bar{T}}{d\bar{P}} = \frac{P_{max}-P_{min}}{T_{max}-T_{min}} \frac{dT_{CR}}{dP_{comb}}$

$$= \frac{P_{max}-P_{min}}{T_{max}-T_{min}} \frac{d}{dP_{comb}}\left(M\sqrt{\left(\frac{1}{P_{comb}}-\zeta\right)}\right)$$

$$= \frac{-M}{2\Delta P_{comb}^2\sqrt{\frac{1}{P_{comb}}-\zeta}} \tag{13}$$

Where, $\Delta = \frac{T_{max}-T_{min}}{P_{max}-P_{min}}$

Now the equation (12) can be written as follows

$$W_2 = \frac{1}{1 + \frac{M}{2\Delta P_{comb}^2\sqrt{\frac{1}{P_{comb}}-\zeta}}} \tag{14}$$

As the summation of all the weights is 1(one) and there are only two objectives in this optimization process, the other weight can be written in the following way.

$$W_1 = 1 - W_2$$
$$= 1 - \frac{1}{1 + \frac{M}{2\Delta P_{comb}^2\sqrt{\frac{1}{P_{comb}}-\zeta}}} \tag{15}$$

Combining the equations (8), (11), (14) and (15) the objective function, Z, presented in equation (7) can be expressed as

$$Z = \left(1 - \frac{1}{1 + \frac{M}{2\Delta P_{comb}^2\sqrt{\frac{1}{P_{comb}}-\zeta}}}\right)\frac{P_{comb}-P_{min}}{P_{max}-P_{min}} + \left(\frac{1}{1 + \frac{M}{2\Delta P_{comb}^2\sqrt{\frac{1}{P_{comb}}-\zeta}}}\right)\frac{M\sqrt{\left(\frac{1}{P_{comb}}-\zeta\right)}-T_{min}}{T_{max}-T_{min}}$$

$$= \frac{P_{comb}-P_{min}}{P_{max}-P_{min}} + \frac{1}{1 + \frac{M}{2\Delta P_{comb}^2\sqrt{\frac{1}{P_{comb}}-\zeta}}}\left(\frac{M\sqrt{\left(\frac{1}{P_{comb}}-\zeta\right)}-T_{min}}{T_{max}-T_{min}} - \frac{P_{comb}-P_{min}}{P_{max}-P_{min}}\right)$$

$$= \frac{1}{P_{max}-P_{min}}\left[P_{comb}-P_{min} + \frac{1}{1 + \frac{M}{2\Delta P_{comb}^2\sqrt{\frac{1}{P_{comb}}-\zeta}}}\left(\frac{M\sqrt{\left(\frac{1}{P_{comb}}-\zeta\right)}-T_{min}}{\frac{T_{max}-T_{min}}{P_{max}-P_{min}}} - P_{comb}+P_{min}\right)\right]$$

$$= \frac{1}{P_{max}-P_{min}}\left[P_{comb}-P_{min} + \frac{1}{1 + \frac{M}{2\Delta P_{comb}^2\sqrt{\frac{1}{P_{comb}}-\zeta}}}\left(\frac{M\sqrt{\left(\frac{1}{P_{comb}}-\zeta\right)}}{\Delta} - \frac{T_{min}}{\Delta} - P_{comb}+P_{min}\right)\right]$$

$$Z=\frac{1}{P_{max}-P_{min}}\left[P_{comb}-P_{min}+\frac{\frac{M}{\Delta}\sqrt{\left(\frac{1}{P_{comb}}-\zeta\right)}-P_{comb}+C}{1+\frac{M}{2\Delta P_{comb}^{2}\sqrt{\frac{1}{P_{comb}}-\zeta}}}\right] \tag{16}$$

Where, $C = P_{min} - \frac{T_{min}}{\Delta}$

The equation (16) is the analytical expression for the objective function of simultaneous AC-DC power flow where the impacts of the loadability and stability both are incorporated.

To get the optimum point or maximum value of the objective function, Z, the equation (16) need to be differentiated with respect to P_{comb} and equated it to zero. i.e. $\frac{dZ}{dP_{comb}}=0$.

Now,

$$\frac{dZ}{dP_{comb}} = 1+\frac{\left(1+\frac{M}{2\Delta P_{comb}^{2}\sqrt{\frac{1}{P_{comb}}-\zeta}}\right)\left(\frac{-M}{2\Delta P_{comb}^{2}\sqrt{\frac{1}{P_{comb}}-\zeta}}-1\right)-\left(\frac{M}{\Delta}\sqrt{\frac{1}{P_{comb}}-\zeta}-P_{comb}+C\right)\frac{d}{dP}\left(\frac{M}{2\Delta P_{comb}^{2}\sqrt{\frac{1}{P_{comb}}-\zeta}}\right)}{\left[1+\frac{M}{2\Delta P_{comb}^{2}\sqrt{\frac{1}{P_{comb}}-\zeta}}\right]^{2}} = 0$$

Or

$$-\left(1+\frac{M}{2\Delta P_{comb}^{2}\sqrt{\frac{1}{P_{comb}}-\zeta}}\right)\left(1+\frac{M}{2\Delta P_{comb}^{2}\sqrt{\frac{1}{P_{comb}}-\zeta}}\right)-\left(\frac{M}{\Delta}\sqrt{\frac{1}{P_{comb}}-\zeta}-P_{comb}+C\right)\frac{d}{dP_{comb}}\left(\frac{M}{2\Delta P_{comb}^{2}\sqrt{\frac{1}{P_{comb}}-\zeta}}\right) = -\left(1+\frac{M}{2\Delta P_{comb}^{2}\sqrt{\frac{1}{P_{comb}}-\zeta}}\right)^{2}$$

Or

$$\left(\frac{M}{\Delta}\sqrt{\frac{1}{P_{comb}}-\zeta}-P_{comb}+C\right)\frac{d}{dP_{comb}}\left(\frac{M}{2\Delta P_{comb}^{2}\sqrt{\frac{1}{P_{comb}}-\zeta}}\right) = 0 \tag{17}$$

The solution of equation (17) can be obtained in the following way

$$\frac{M}{\Delta}\sqrt{\frac{1}{P_{comb}}-\zeta}-P_{comb}+C=0 \tag{18}$$

$$or \quad P_{comb}-C = \frac{M}{\Delta}\sqrt{\frac{1}{P_{comb}}-\zeta}$$

$$or \quad (P_{comb}-C)^{2}=\left(\frac{M}{\Delta}\right)^{2}\left(\frac{1}{P_{comb}}-\zeta\right)$$

$$or \quad P_{comb}^{2}-2P_{comb}C+C^{2}=K\left(\frac{1-\zeta P_{comb}}{P_{comb}}\right) \tag{19}$$

Where,

$$K=\left(\frac{M}{\Delta}\right)^{2}$$

Equation (19) can be rearranged as

$$P_{comb}^{3}-2P_{comb}^{2}C+C^{2}P_{comb}=K-K\zeta P_{comb}$$

$$P_{comb}^{3}+(-2C)P_{comb}^{2}+(K\zeta+C^{2})P_{comb}+(-K)=0 \tag{20}$$

Now the solution of equation (20) is as follows:

$$P_{comb}=\frac{1}{3}\left(2C-R-\frac{D_0}{R}\right) \tag{21}$$

Where,

$$R = \sqrt[3]{\frac{D_1 + \sqrt{D_1^2 - 4D_0^3}}{2}}$$

$$D_0 = C^2 - 3K\zeta$$

$$D_1 = 2C^3 + 18CK\zeta - 27K$$

The equation (21) is the analytical expression for combined power flow for which the value of the objective function would be maximum i.e. both the objectives, loadability and stability, can be improved optimally.

3. NUMERICAL ANALYSIS

The numerical analysis of the objective function and its optimal point are presented in this section. It is true that there is no scope to achieve very high amount of combined improvement of loadability and stability. More precisely, it can be said that if someone want to achieve very high amount of loadability improvement he has to lose the stability improvement and vice versa. In this regard, to clarify the proposed model a typical single circuit transmission system is considered in this section which is presented in Figure 1.

This power system has a long transmission line with a generation capacity of 1100MVA. The length of the line is 400km and the voltage of the line is 345kV. Basically this transmission system is evacuating power from 132kV generator bus to a distant infinite bus of 132kV. Now the the pure AC system is converted into simultaneous AC-DC system and it is shown in Figure 2. The the numerical values of all the parameters from generator to infinite bus for pure AC and for simultaneous AC-DC system are presented in Appendix-A.

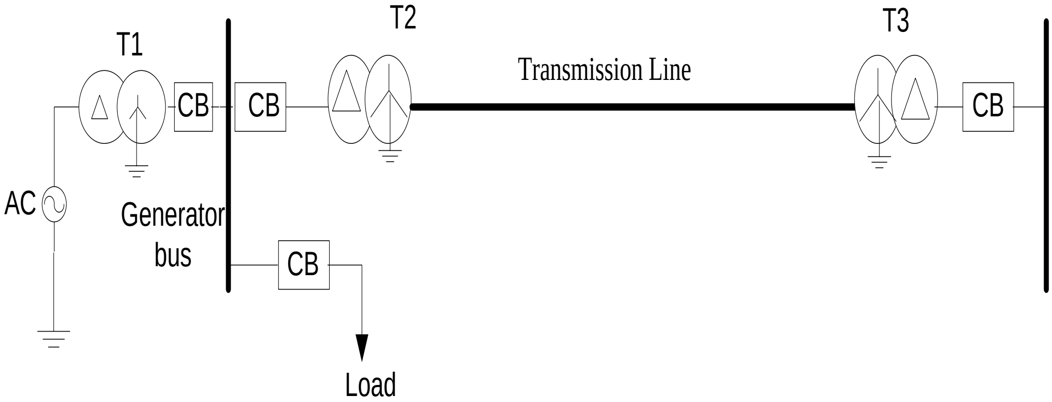

Figure 1.Single circuit AC power transmission system

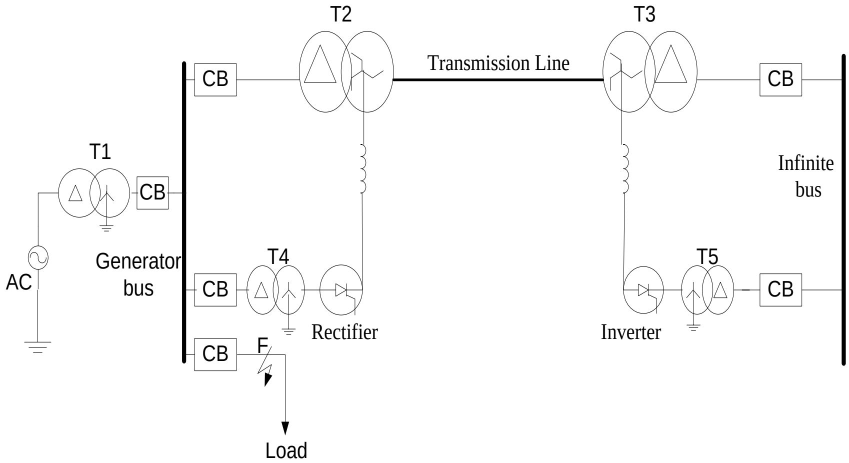

Figure 2. Simultaneous AC-DC power transmission system

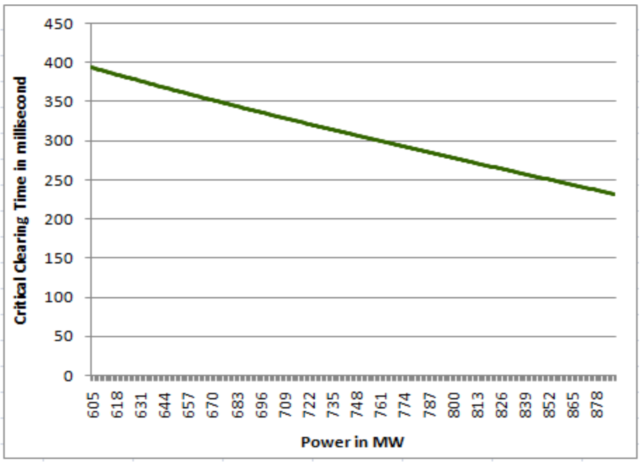

Figure 3. Stability versus loadability

Using the numerical data presented in Appendix-A, initially the loadability and stability analysis is performed for pure AC system. The steady state loadability of pure AC system is found as 605MW considering 30% steady state stability margin. Incase of stability analysis a 3-phase to ground fault is considered at the load terminal and obtained critical clearing time (CCT) is of 231ms. Considering 605MW and 231ms as base values for the operation of simultaneous AC-DC system the stability analysis is performed for the same type of fault with different values of steady state power. In this case, obtained CCT is of 380ms for 605MW of steady state loading and 231ms for 898MW of steady state loading. The detail analysis is shown in Figure 3.

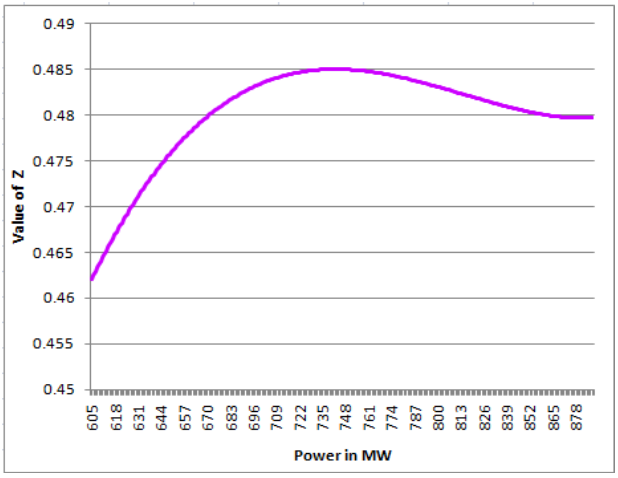

Figure 4. Objective function variations with respect to combined power flow

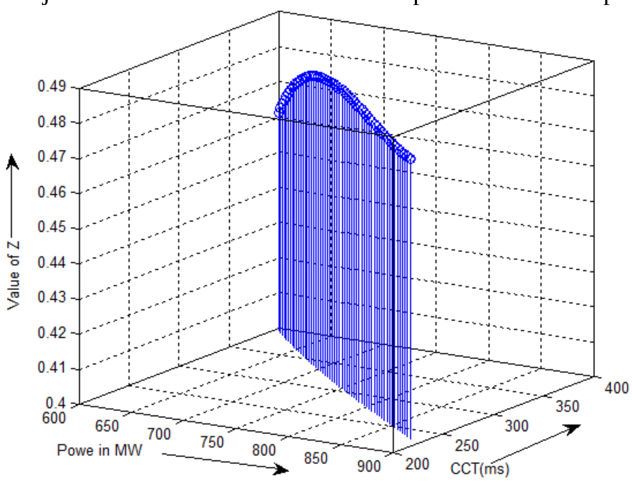

Figure 5. Objectives function with respect to power flow and CCT

It is seen from the Figure 3 that the minimum loadability point gives maximum stability (CCT) and maximum loadability point gives minimum stability. To obtain the improvements in both loadability and stability in the considered system at a time the analytical expression (16) is used and the objective function, Z, found from the numerical data is shown in Figure 4. The objective function Z is mainly the combined improvement of loadability and stability. In this figure the variation of objective function is shown with respect the variation in combined power flow. It is clearly observed that the value of Z is initially increasing in nature with the increase of combined power flow and for a particular value of combined power it has a steady state value and further increase of combined power flow gives the decreasing value of Z. As the objective is to find the maximum value of the objective function Z the steady state value of Z in Figure 4 is the maximum value. To get an overall picture of the objective function with respect to both the decision variables; loadability and stability, a 3-D plot is presented in Figure 5.

The main objective of this optimization process in this paper is to find the value of combined power flow for which the objective function would be maximum. Applying the equation (21) in this considred system it is found that the maximum value of the objective function of this particular system occurs at 742MW of loading and at this loading the CCT of the system is is of 313ms. This point is actually the desired optimal operating point of this system where the objective function is maximum. Therefore, it can be said that at the optimal operating point of this particular system the loadability and stability improvements are 22.65% and 35.5% respectively.

To investigate the impacts of the line voltage and line length of a transmission system on this optimal point two different analysis are performed. One, changing the voltage of this considred system keeping the other parameters unchanged and other one, changing the line length keeping the other parameters same as their original value. The impact of voltage change on the optimal point is presented in Figure 6.

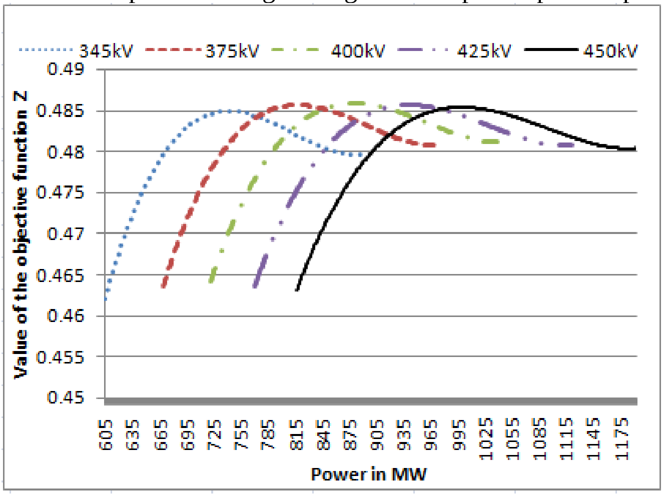

Figure 6. Impact of voltage change on the optimal point

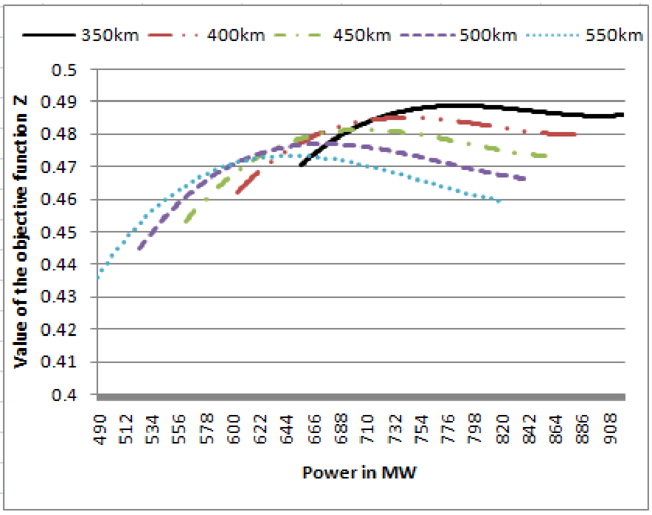

Figure 7. Impact of the line length on the optimal point

Figure 6 clearly reveals that the optimal point shifted from lower loadability to higher loadability with the increase of voltage level of the system. That is, a system with higher voltage level has higher power carrying capacity at its optimal operating point and a system with lower voltage level has the lower power carrying capacity at its optimal operating point point.

The impact of the change of line length on the optimal point is presented in Figure 7. It is clearly observed that the optimal points moving towards the lower loadability from the higher loadability with the increase of line length of the system. That is, the longer the transmission line the lower the load carrying capability at its optimal point.

Now, using equation (21) the power flow at the optimal operating point for the considered system, changing the voltage level and line length, are evaluated and presented in Table 1. The 3rd and 6th columns of the table present the power flow at the optimal operating point of simultaneous AC-DC systemand the 2nd and 5th columns of the table also show the power flow through pure AC system for different voltage level and line length respectively. Note that the power flow in pure AC system is calculated considering the recommended loadability margin for the long transmission line [1].

Table 1. Optimal Power Flow in Comparison With Original AC Power Flow

Change of voltage level			Change of line length		
Voltage (kV)	Power flow in pure AC system (MW)	Power flow in AC-DC system at the optimal point (MW)	Line length (km)	Power flow in pure AC system (MW)	Power flow in AC-DC system at the optimal point(MW)
345	605	742	350	657	787
375	670	818	400	605	742
400	723	882	450	562	708
425	771	942	500	524	676
450	818	100	550	490	648

From Table 1 it is clearly observed that the power flow at the optimal operating point of simultaneous AC-DC system at any voltage level and line length is much higher than that of its pure AC system. Although the stability margins (CCTs) at the optimal operating point of the simultaneous AC-DC system have not been mentioned in the table but there will be significant increase in stability margin.

The improvements of loadability and stability of simultaneous AC-DC system at the optimal points, in comparison with that of pure AC system, for different line length and voltage level are presented in Figures 8 and 9 respectively. It is seen that the stability improvement (SI) is higher than the loadability improvement (LI) at the optimal points for the change of voltage and line length both. In case of change in line length the LI and SI both are increasing in nature with the increase in the length of transmission line. On the contrary, the LI and SI both have approximately constant magnitude at the optimal points for all voltage levels. It is also seen from Figure 8 that the difference between the magnitude of LI and SI decreases with the increase of line length.

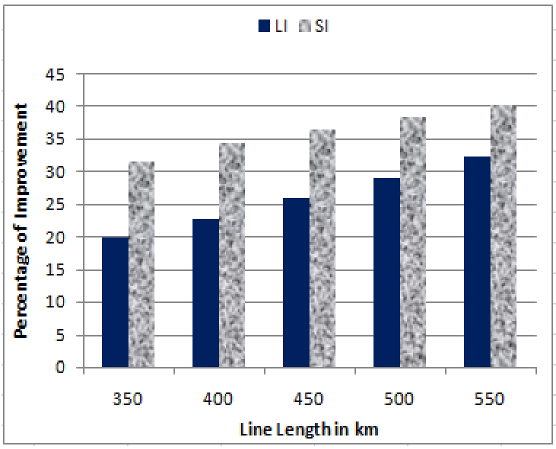

Figure 8. Comparison of LI and SI for the change of line length at the optimal point

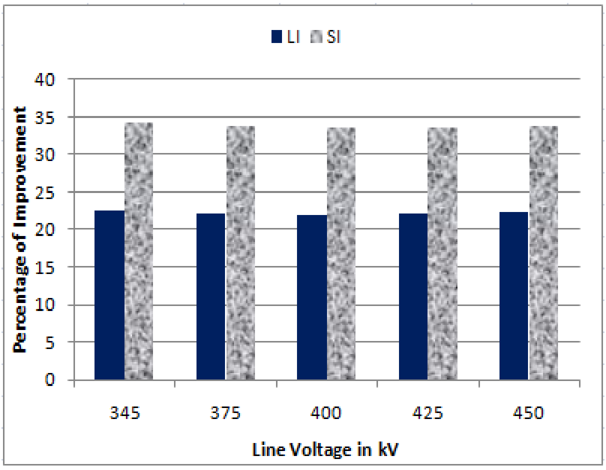

Figure 9. Comparison of LI and SI for the change of line voltageat the optimal point

4. VALIDATION OF THE DEVELOPED MODEL

The mdel which developed for the optimal operation of simultaneous AC-DC system in section 2 is validated in this section. A power system with its numerical data is considered for this validation. The validation is performed comparing the results obtained through developed model with those obtained through MATLAB simulation. The considered power system is presented in Figure 10.

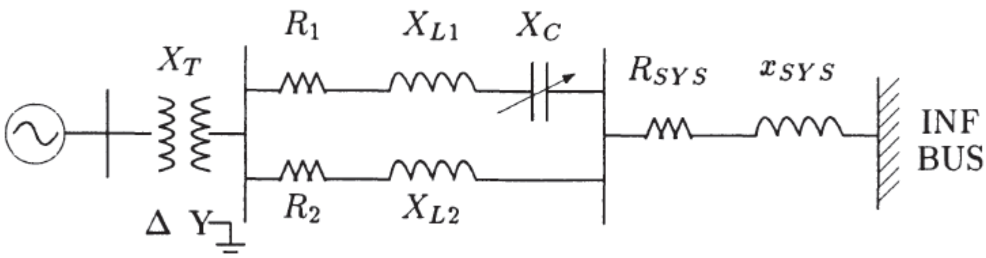

Figure 10. IEEE second benchmark system

The system presented in Figure 10 is a 500kV double circuit transmission system. The system parameters are presented in Table 2. In this analysis all the resistances and capacitive compensation of the system are neglected.

Table 2. Network impedance in per unit based on 100 MVA

Parameter	Positive sequence	Zero sequence
R_T	0.0002	0.0002
X_T	0.02	0.02
R_1	0.0074	0.022
X_{L1}	0.08	0.24
R_2	0.0067	0.0186
X_{L2}	0.0739	0.21
R_{sys}	0.0014	0.0014
X_{sys}	0.03	0.03

The AC system is converted into simultaneous AC-DC system and initially MATLAB simulation is performed for the optimal operation of the system and then the developed model is applied to the system. The results of MATLAB simulation and developed model are compared and presented in Table 3 and Table 4. The comparison is shown in Table 3 and Table 4 considering different DC voltage level and different transmission angle respectively.

Table 3. Results comparison between MATLAB Simulation and developed model with changing the DC voltage level

% of DC voltage mix	Loadability			Stability		
	Through developed model	Through MATLAB simulation	Difference (%)	Through developed model	Through MATLAB simulation	Difference (%)
20	842MW	846MW	0.47	175.86 ms	175.02 ms	-0.48
30	928MW	931MW	0.32	179.22 ms	178.71 ms	-0.28
49.5	1050MW	1051MW	0.095	182.3ms	182.14ms	-0.087

It is clearly observed from Table 3 and Table 4 that the loadability and stability obtained through developed model are very much close to those obtained from MATLAB simulation at the optimal operating point. The tables also reveal that the magnitude of differences between these two approaches in case of loadability and stability are less than 0.5% which indicates the acuuracy level of the model is extremely high.

Table 4. Results comparison between MATLAB Simulation and developed model with changing the transmission angle

Transmission angle	Loadability			Stability		
	Through developed model	Through MATLAB simulation	Difference (%)	Through developed model	Through MATLAB simulation	Difference (%)
20	1100 MW	1101 MW	0.09	191.23 ms	191.1 ms	-0.068
30	1081 MW	1081 MW	0	187.49 ms	187.49 ms	0
50	1036 MW	1037 MW	0.096	179.9 ms	179.7 ms	-0.11

5. APPLICATION OF THE ANALYTICAL MODEL OF OPTIMAL POINT

To judge the applicability of the proposed model a 500kV, 804km long transmission system is considered. In this transmission system power is evacuated from Colstrip (eastern Montana) to Taft. The Montana generation system has four generating units with a total capacity of 2272MW. The circuit model of the system is presented in Figure 11[22] anda detail description of the system is presented in Appendix-B.

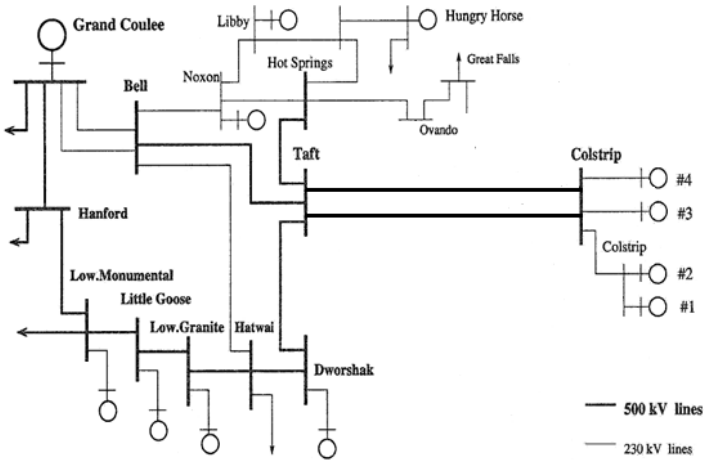

Figure 11. Montana 500kV transmission system

Firstly, the analysis of pure AC system is performed for the considered system. In case of pure AC system the loadability is evaluated considering the steady state stability margin of 30% which is the recommended stability margin for long transmission line [1]. In this case the evaluated loadability is 1028MW. The stability (CCT) of pure AC system is evaluated considering a severe most fault (3-phase to ground) at the Colstrip generator bus with 1028MW of pre-fault steady state loading and the obtained CCT is 175ms.

Secondly, the AC system is converted into simultaneous AC-DC system and then the developed expression, equation (21), is applied for the optimal operation of the system. In this case the loadability and stability of the system at the optimal points are evaluated considering the variation of AC and DC voltage mix and transmission angle. As the simultaneous AC-DC transmission system contains AC and DC both form of voltage in the same line it is an opportunity to operate the system by changing the magnitude of AC and DCvoltages. Moreover, this AC-DC system gives another opportunity to operate the transmission system with wide range of variation of transmission angle.

The loadability and stability improvements for the optimal operation of the simiulaneous AC-DC system are evaluated in comparison with the loadability and stability of pure AC system. The evaluated improvements in loadability and stability at the optimal operating points are presented in Table 5 considering the variation of voltage mix of AC-DC system. During the variation of AC and DC voltage mix a constant transmission angle of 44.47^0 of AC power flow is considered.

Table 5. Improvement of Loadability and Stability at the Optimal Point for Different Voltage Mix

% of voltage mix		Power flow at the optimal point (MW)	Loadability improvement with optimal operation (%)	Stability improvement with optimal operation (%)
V_{ac}	V_{dc}			
90	10	1158	12.65	18.29
80	20	1319	28.31	38.86
70	30	1450	41.04	44.57
60	40	1567	52.43	48.00
50.5	49.5	1664	61.87	49.71

It is observed that the loadability and stability improvements at the optimal points increase with the increase of DC voltage mix. The table also reveals that the rate of increase in loadability improvement is higher than that of stability improvement with the increase of DC voltage mix. At the highest point of DC voltage mix (49.5%) the optimal operation gives 61.87% and 49.71% of loadability and stability improvements respectively. Note that, in simultaneous AC-DC system the DC voltage mix must be less than 50% of original AC voltage [15].

Table 6 presents the variation of loadability and stability improvements with the variation of transmission angle at the optimal operating points of simultaneous AC-DC system. In case of the variation of transmission angle a constant DC voltage mix of 49.5% is considered for the operation of the system. It is also found that the improvements are decreasing in nature with the increase of transmission angle. At the lowest level of transmission angle (20 degree) in the table the optimal point gives 68.48% and 57.71% of loadability and stability improvements respectively.

Table 6. Improvement of Loadability and Stability at the Optimal Point for Different Transmission Angle

Power transmission angle (degree)	Power flow at the optimal point	Loadability improvement with optimal operation (%)	Stability improvement with optimal operation (%)
20	1732	68.48	57.71
30	1706	65.95	54.29
40	1678	63.23	51.43
50	1647	60.21	48.00
60	1613	56.91	44.57

6. CONCLUSION

Loadability and stability are reciprocal in nature. That is, increase in one parameter causes decrease in other parameter. Simultaneous AC-DC power flow can increase loadability and stability both at the same time but maintaining a trade-off between these two. This paper presents an analytical expression for the combined improvement of loadability and stability in case of simultaneous AC-DC system. Applying this expression in any system an optimal point can be found where loadability and stability both improvement can be achieved at a reasonable level.

Through numerical analysis it is seen that the objective function curve has a maximum point. Initially the value of the objective function increases with the increase of power flow and after reaching a certain point it starts decreasing with the increase of power flow.

The numerical analysis for different voltage and line length also clearly reveal that the optimal points shifted from lower loadability to higher loadability with the increase of voltage level while maintaining approximately constant magnitude of the improvements in loadability and stability at the optimal points. On the contrary, the optimal points shifted from higher loadability to lower loadability with the increase of line length while maintaining a significant increase in the magnitude of the loadability and stability improvements at the optimal points.

The accuracy of the developed model is extremely high which has been established in the validation section by comparing the results with the MATLAB simulation. From the application of the model it is seen that the improvements in loadability and stability at the optimal points are found in increasing and decreasing in nature with the increase of DC voltage mix and transmission angle, respectively. Note that, the impact of the change of voltage mix is much higher than the change in transmission angle on the loadability and stability improvements at the optimal points.

APPENDIX

Appendix-A

The parameters of different components of the power system considered for the numerical analysis of the proposed model is presented in Table A.

Table A. Parameters of the Different Components of Single Circuit Transmission System

Sl. No.	Component	Parameter
01	Line	z = 0.01755 +j0.3292 Ω/km/phase Single ckt, Three phase, 60Hz, 400km, 345kV, Thermal limit current = 1.8kA, ACSR twin bundle conductor
02	Generator	1100MVA, 24kV, 60Hz, the parameters on its own base – Xd =1.305 , Xd´ =0.3 , Xd″ =0.3 , Xq =0.474 , Xq″=0.243 , X =0.18 , Stator resistance Rs=0.00285 , Td´ =1.01s, Td″ =0.053s , Tqo″ =0.1s ,
03	Generator Transformer	1100MVA, 24/132 kV, 60Hz, 10% reactance.
04	Transformer (At the sending end of the line):	Δ-Y, 1100MVA, 132/345kV, 60Hz, 16% reactance. (pure AC) Δ-Z, 500MVA, 132/172.5kV, 60Hz, 16% reactance. (AC-DC)
05	Transformer (At the receiving end of the line):	Y -Δ 1100MVA, 345/132kV, 60Hz, 16% reactance.(Pure AC) Z -Δ 500MVA, 172.5/132kV, 60Hz, 16% reactance (AC-DC)
06	DC system	The Rectifier and Inverter are 12-pulse converters using two 6-pulse thyristor bridges connected in series, DC current (rated)=5.4kA, Smoothing reactor=0.5H, Rectifier firing angle(minimum)=5^0, Inverter Extinction angle(minimum)=14^0.

Appendix-B

For the application of the proposed model only 500kV double circuit transmission line from Colstrip to Taft is considered, excluding other portion of the power system [22]. Colstrip with the generation capacity of 2272MW is considered as an equivalent machine (generator) and Taft is considered as an infinite bus. The application also does not consider Broadview and Garrison buses in between Colstrip and Taft. The detail parameters are presented in Table B.

Table B.The Parameters of Montana 500kV Transmission System

Sl. No.	Component	Parameter
01	Line	x = j253.21 Ω/phase/ckt, Double ckt, Three phas, 60Hz, 804km'500kV,Thermal limit current = 3kA
02	Generator	358*2, 778*2(MW) 24kV,Reactance = 0.3pu, H=3.5 s.
03	Generator Transformer	24/230kV, Leakage reactance = 0.15pu.
04	Transformer (At the sending end of the line):	Δ-Y, 230/500kV, leakage reactance = 0.1pu. (pure AC) Δ-Z, 230/253kV, Leakage reactance = 0.1pu. (AC-DC)
05	Transformer (At the receiving end of the line):	Z-Δ 253/500kV, Leakage reactance= 0.1pu, (AC-DC)
06	DC system	DC system rated voltage and current are 202kVand 9kA, respectively.

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BIOGRAPHIES OF AUTHORS

	**Mohammad TawhidulAlam**was born in Bangladesh in 1977**.** Hereceived B.Sc. (Engg.) degree from Dhaka University of Engineering and Technology (DUET) in 2002, and M.Sc. (Engg.) degree from Bangladesh University of Engineering and Technology(BUET) in 2008. He obtained his Ph.D. degree in 2016 in the department of Electrical and Electronic Engineering at Bangladesh University of Engineering and Technology, Dhaka, Bangladesh. Dr. Alam served as a faculty member at Bangladesh College of Leather Technology from 2003 to 2008 and at Ahsanullah University of Science and Technology (AUST) from 2008 to 2018. Currently, he is serving as anAssociate Professor in the Department of Electrical and Electronic Engineering at Daffodil International University, Dhaka, Bangladesh. His research interests include HVDC, power system stability and renewable energy.
	Q. Ahsan was born in Bangladesh in 1951. He graduated from Bangladesh University of Engineering and Technology (BUET), Dhaka in 1975. Dr. Ahsan did his M. Sc. Engineering from the same university in 1980 and Ph. D from the University of Ottawa, Canada in 1984. Dr. Ahsan joined in the Department of Electrical and Electronic Engineering of BUET as a lecturer in 1976 and he became a full professor in 1988. He has worked as a visiting faculty in a number of Universities at home and abroad. Dr. Ahsan has also worked as a consultant of many important national and international projects. He has published around 90 technical papers in the reputed national and international journals and conferences. He is the author of couple of books and a chapter of a book. He is the recipient of a number of awards.